CH 0 – PROLOGUE

❑ *THE REAL NUMBERS*

The term **real number** refers to any number that is a **decimal**, or can be written as a decimal.

Examples of Real Numbers

14	14 is written 14.0
7.45	It already is a decimal.
$-\frac{4}{5}$	Same as –0.8
0.757575...	It already is a decimal.
π	3.14159. . .
$-\sqrt{2}$	–1.41421 . . .
$\frac{2}{3}$	0.666 . . .

So, what's NOT a *real number*? The classic example is

$$\sqrt{-1}$$

We ask ourselves: What number, when squared, equals –1? No way! Every number squared is at least 0, and most likely positive, never negative.

We thus have a number, $\sqrt{-1}$, that cannot be written as a decimal, and therefore is not a real number.

But we do call it an **imaginary number** denote it with the letter *i*.

$$i = \sqrt{-1}$$

Homework

1. Explain why each of the following numbers is classified as a **real number**:

   a. 34.56 b. -99 c. $\frac{\pi}{3}$ d. $\sqrt{44}$ e. $\sqrt{25}$

   f. $\frac{4}{5}$ g. $\frac{23}{99}$ h. $0.010010001\ldots$

   i. 2.74365 j. $-44.90867633\ldots$ k. 0 l. $\sqrt[3]{-64}$

2. Explain why each of the following numbers is not a real number:

   a. $\sqrt{-9}$ b. $\sqrt{-30}$ c. $\sqrt[4]{-16}$

❑ *TWO IMPORTANT THINGS WE DO TO REAL NUMBERS*

Opposite

The ***opposite*** of a real number is found by changing the sign of the number. For example, the opposite of 7 is -7, the opposite of $-\pi$ is π, and the opposite of 0 is 0 (since 0 doesn't really have a sign). The opposite of n is $-n$, and the opposite of $-n$ is n. Also notice that the sum of a number and its opposite is always 0; for example, $17 + (-17) = 0$.

When considering numbers on the real number line, two numbers are opposites of each other if they're the same distance from 0, but on opposite sides of 0. [Note that although 0 is the opposite of 0, it's kind of hard to justify the claim that they're on "opposite" sides of 0.]

Homework

3. What is the **opposite** of each number?

   a. 17 b. 0 c. -3.5 d. 8π e. $-\sqrt{2}$

4. a. T/F: Every number has an opposite.

   b. The opposite of 0 is ____.

   c. The opposite of a negative number is always _____.

   d. The opposite of a positive number is always _____.

   e. The sum of a real number and its opposite is always _____.

5. Using the formula $y = -x$, find the y-value for the given x-value:

   a. $x = 9$ b. $x = -3$ c. $x = 0$ d. $x = \pi$ e. $x = -\sqrt{2}$

Reciprocal

The ***reciprocal*** of a real number is found by dividing the number into 1. Equivalently, the reciprocal of x is $\frac{1}{x}$, and the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$. Every real number has a reciprocal <u>except</u> 0; the reciprocal of 0 would be $\frac{1}{0}$, which is undefined, as explained in detail later in this Prologue.

Notice that the reciprocal of a positive number is positive, and the reciprocal of a negative number is negative. In addition, **the product of any real number with its reciprocal is always 1**; for example, $\frac{2}{7}\cdot\frac{7}{2} = 1$.

Homework

6. Find the **reciprocal** of each real number:

   a. 5 b. $\frac{2}{9}$ c. $-\frac{7}{3}$ d. 1 e. 0 f. $\frac{1}{\pi}$ g. $-\sqrt{3}$

7. a. T/F: Every number has a reciprocal.

   b. The reciprocal of 0 is ____.

   c. The reciprocal of a negative number is always _____.

   d. The reciprocal of a positive number is always _____.

   e. The product of a real number and its reciprocal is always ____.

8. Using the formula $y = \frac{1}{x}$, answer each question:

   a. If $x = 14$, then $y =$ ____.

   b. If $x = \frac{2}{3}$, then $y =$ ____.

   c. If $x = -99$, then $y =$ ____.

   d. If $x = -\frac{5}{4}$, then $y =$ ____.

   e. If $x = 0$, then $y =$ ____.

❑ *THE FIVE LAWS OF EXPONENTS*

A. $x^3x^4 = x^7$

B. $\frac{x^{10}}{x^2} = x^8$

C. $(xy)^5 = x^5y^5$

D. $\left(\frac{x}{y}\right)^{10} = \frac{x^{10}}{y^{10}}$

E. $(x^3)^4 = x^{12}$

F. $x^0 = 1$ (as long as $x \neq 0$)

G. $\frac{w^{12}}{w^{15}} = \frac{1}{w^3}$

H. $xy^0 + (xy)^0 = x(1) + 1 = x + 1$

I. a^3b^4 cannot be simplified (the bases are not the same).

Homework

9. Simplify each expression:

a. x^7x^4 b. $\dfrac{y^4}{y^3}$ c. $(r^4)^5$ d. $(ab)^{12}$

e. $\left(\dfrac{p}{q}\right)^4$ f. $a^3a^3a^2$ g. $(abw)^3$ h. $(2h)^0 + 2h^0$

❑ *FRACTIONS*

Operations with Fractions

$$-\frac{2}{3}-\frac{1}{2} = -\frac{4}{6}-\frac{3}{6} = -\mathbf{\frac{7}{6}}$$

$$\frac{4}{5}-\left(-\frac{2}{3}\right) = \frac{4}{5}+\frac{2}{3} = \frac{12}{15}+\frac{10}{15} = \mathbf{\frac{22}{15}}$$

$$\frac{2}{9}-7 = \frac{2}{9}-\frac{63}{9} = -\mathbf{\frac{61}{9}}$$

$$\left(\frac{2}{3}\right)\left(-\frac{5}{7}\right) = -\mathbf{\frac{10}{21}}$$

$$-\frac{4}{7}\div -2 = -\frac{4}{7}\times -\frac{1}{2} = \frac{4}{14} = \mathbf{\frac{2}{7}}$$

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)\left(-\frac{1}{4}\right)\left(-\frac{1}{5}\right) = \mathbf{\frac{1}{120}}$$

$$\frac{-8}{\frac{1}{3}} = -\frac{8}{1}\div\frac{1}{3} = -\frac{8}{1}\times\frac{3}{1} = -\mathbf{24}$$

$$\frac{-\frac{9}{4}}{-2} = -\frac{9}{4}\div -2 = -\frac{9}{4}\div -\frac{2}{1} = -\frac{9}{4}\times -\frac{1}{2} = \mathbf{\frac{9}{8}}$$

Note: A negative sign can "float." For instance,

$$\frac{-30}{6} = \frac{30}{-6} = -\frac{30}{6}$$

since all of these fractions have the value –5.

Powers and Square Roots of Fractions

An **exponent** still means what it always has, so these next examples should be clear.

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3}\cdot\frac{2}{3} = \mathbf{\frac{4}{9}} \qquad \left(-\frac{1}{4}\right)^3 = \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right) = -\mathbf{\frac{1}{64}}$$

$$\left(-\frac{9}{4}\right)^1 = -\mathbf{\frac{9}{4}} \qquad \left(\frac{1}{2}\right)^8 = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \mathbf{\frac{1}{256}}$$

As for the **square root sign**, we still ask: What number (that's not negative) times itself gives the number in the radical sign?

$\sqrt{\frac{9}{25}} = \mathbf{\frac{3}{5}}$ This is true because $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$.

$\sqrt{\frac{1}{144}} = \mathbf{\frac{1}{12}}$ This is due to the fact that $\frac{1}{12}\times\frac{1}{12} = \frac{1}{144}$.

$\sqrt{-\frac{4}{49}}$ does not exist as a real number, because $-\frac{4}{49}$ is a negative number, and square roots of negative numbers are outside the real numbers. It's an imaginary number.

$\sqrt{\frac{-4}{-49}}$ does exist as a real number, because the fraction is actually a positive number: $\sqrt{\frac{-4}{-49}} = \sqrt{\frac{4}{49}} = \mathbf{\frac{2}{7}}$.

Homework

Perform the indicated operation:

10. a. $-\frac{1}{2}-\frac{4}{5}$ b. $-\frac{1}{3}-\left(-\frac{1}{3}\right)$ c. $\frac{2}{3}-\left(-\frac{5}{6}\right)$

d. $-\frac{4}{5}+\frac{2}{3}$ e. $9-\frac{4}{5}$ f. $-1-\frac{2}{3}$

g. $\frac{8}{3}-5$ h. $-\frac{2}{3}-(-1)$ i. $-\frac{1}{4}-\frac{2}{7}$

11. a. $\left(-\frac{1}{2}\right)\left(-\frac{5}{6}\right)$ b. $\left(-\frac{2}{3}\right)\left(\frac{3}{2}\right)$ c. $-\frac{5}{6}\cdot-\frac{6}{5}$

d. $-\frac{2}{3}\div-\frac{3}{2}$ e. $\frac{1}{2}\div-9$ f. $7\div-\frac{3}{4}$

g. $\dfrac{-\frac{2}{3}}{-\frac{1}{9}}$ h. $\dfrac{\frac{4}{5}}{-8}$ i. $\dfrac{-\frac{4}{5}}{\frac{5}{8}}$

12. True/False: $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$ [assuming $b \neq 0$]

13. a. $\left(-\frac{2}{3}\right)^2$ b. $\left(-\frac{1}{2}\right)^3$ c. $\left(-\frac{1}{3}\right)^4$

d. $\left(-\frac{14}{19}\right)^1$ e. $\left(-\frac{1}{2}\right)^5$ f. $\left(-\frac{2}{3}\right)^6$

14. a. $\sqrt{\frac{81}{100}}$ b. $\sqrt{\frac{36}{64}}$ c. $\sqrt{\frac{1}{4}}$

d. $\sqrt{\frac{1}{9}}$ e. $\sqrt{\frac{121}{144}}$ f. $\sqrt{-\frac{25}{81}}$

g. $\sqrt{\frac{-256}{-289}}$ h. $\sqrt{-\frac{14}{17}-\left(-\frac{14}{17}\right)}$

❑ *ORDER OF OPERATIONS*

Order of Operations
Parentheses and Brackets [( )]
Exponents
Multiply & Divide (left to right)
Add & Subtract (left to right)

Note: Certainly $(-5)^2 = 25$, since both the 5 and the minus sign are being squared [i.e., $(-5)^2 = (-5)(-5) = 25$]. However, consider the expression

$$-5^2$$

Do we square the –5? The answer is NO; the exponent attaches to the 5 only. The justification is the Order of Operations, which states that exponents (near the top of the chart) are to be done before we deal with negative signs (which are at the bottom of the chart). So, although $(-5)^2 = 25$, we must agree that

$$-5^2 = -25$$

Homework

15. Evaluate (simplify) each expression:

a. $3 \cdot 10^2 - (8 - 4)^3 - 3 \times 2$

b. $(5 - 3)^2 + (10 - 7)^3$

c. $[3 + 2(5)] - 1 + 3 \cdot 10$

d. $2(10 - 5)^2 - 12 \div 3$

e. $[2(10 - 5)]^3 \div (10 \cdot 10^2)$

f. $(1 + 4)^2 - (4 + 1)^2$

g. $[(3^2 - 2^2)^3 - 80] \div (36 / 4)$

h. $3 \cdot 4^2 - (13 - 12)^3$

i. $10 + 8(8 - 1)^2 - 3 - 2 - 1$

j. $[8^2 - 2^3 + 3 \cdot 4 - 2(7)]^2$

k. $[20 - (5 - 2)^2]^2 - 2 \cdot 3 \cdot 4$

l. $[13 - (8 - 3) + (10 - 2)]^3$

16. Evaluate each expression for the given values:

a. $(x + y)^2$ for $x = 2$ and $y = 1$

b. $x^2 + y^2$ for $x = 10$ and $y = 5$

c. $x^2 + xy + y^2$ for $x = 3$ and $y = 6$

d. $(x + y)(x - y)$ for $x = 10$ and $y = 2$

e. $x^2 - y^2$ for $x = 12$ and $y = 10$

❑ Division and Those Pesky Zeros

The result of dividing by zero

It's a mathematical fact of life that the only number that's never allowed to be in the denominator (bottom) of a fraction is zero. Sometimes this is phrased

"Never divide by zero."

What's the big deal?

Recall from elementary school that

$$\frac{56}{7} = 8 \text{ because } 8 \times 7 = 56.$$

Zero on the Top (but not on the bottom)

How shall we interpret the division problem

$$\frac{0}{7} = ???$$

What number times 7 yields an answer of 0? Well, 0 works; that is,

$$\frac{0}{7} = 0 \quad \text{because } 0 \cdot 7 = 0.$$

Moreover, no other number besides 0 will work.

Zero on the Bottom (but not on the top)

Now let's put a zero on the bottom and see what happens:

$$\frac{9}{0} = ???$$

Let's try an answer of 0; unfortunately $0 \cdot 0 = 0$, not 9.

How about we try an answer of 9? Then $9 \cdot 0$ is also 0, not 9.

Could the answer be π? No; $\pi \cdot 0 = 0$, not 9.

In fact, any number we surmise as the answer will have to multiply with 0 to make a product of 9. But this is impossible, since any number times 0 is always 0, never 9. In short, no number in the whole world will work in this problem.

Zero on the Top AND the Bottom

Now for an even stranger problem with division and zeros:

$$\frac{0}{0} = ???$$

We can try 0; in fact, since $0 \cdot 0 = 0$, a possible answer is 0.

Let's try an answer of 5; because $5 \cdot 0 = 0$, another possible answer is 5.

Could π possibly work? Since $\pi \cdot 0 = 0$, another possible answer is π.

Is there any end to this madness? Apparently not, since any number we conjure up will multiply with 0 to make a product of 0. In short, every number in the whole world will work in this problem.

Summary:

1) Zero on the top of a fraction is perfectly okay, as long as the bottom is NOT zero. The answer to this kind of division problem is always zero. For example, $\frac{0}{7} = 0$.

2) There is no answer to the division problem $\frac{9}{0}$. Clearly, we can never work a problem like this.

3) There are infinitely many answers to the division problem $\frac{0}{0}$. This may be a student's dream come true, but in mathematics we don't want a division problem with trillions of answers.

Each of the problems with a zero in the denominator leads to a major conundrum, so we summarize cases 2) and 3) by stating that

DIVISION BY ZERO IS UNDEFINED!

Our final summary:

$$\frac{0}{7} = 0$$

$$\frac{9}{0} \text{ is undefined}$$

$$\frac{0}{0} \text{ is undefined}$$

"Black holes are where God divided by zero."

Steven Wright

Homework

17. Evaluate each expression, and explain your conclusion:

a. $\frac{0}{15}$ b. $\frac{32}{0}$ c. $\frac{0}{0}$

18. Evaluate each expression:

a. $\frac{0}{17}$ b. $\frac{0}{-9}$ c. $\frac{6-6}{17+3}$ d. $\frac{3^2-8-1}{100}$

e. $\frac{98}{0}$ f. $\frac{-44}{0}$ g. $\frac{7+8}{2^3-8}$ h. $\frac{7^2-40}{-23+23}$

i. $\frac{0}{0}$ j. $\frac{-9+9}{10-10}$ k. $\frac{5^2-25}{0^2+0^3}$ l. $\frac{4\cdot 5-2\cdot 10}{3^3-9}$

19. $\frac{0}{\pi} = 0$ because

  a. 0 is the only number multiplied by π that will produce 0.
  b. no number times π equals 0.
  c. every number times π equals 0.

20. $\frac{0}{0}$ is undefined because

  a. no number times 0 equals 0.
  b. every number times 0 equals 0.
  c. any number divided by itself is 1.

21. $\frac{7}{0}$ is undefined because

  a. 0 is the only number multiplied by 0 that will produce 7.
  b. no number times 0 equals 7.
  c. every number times 0 equals 7.

22. a. The numerator of a fraction is 0. What can you conclude?
  b. The denominator of a fraction is 0. What can you conclude?

❑ LINEAR EQUATIONS AND FORMULAS

<u>Solve for x:</u> $\mathbf{2(3x - 7) - 5(1 - 3x) = -(-4x + 1) + (x + 7)}$

<u>Solution:</u> The steps are

1) Distribute
2) Combine like terms
3) Solve the simplified equation

$2(3x - 7) - 5(1 - 3x) = -(-4x + 1) + (x + 7)$

$\Rightarrow \quad 6x - 14 - 5 + 15x = 4x - 1 + x + 7$ (distribute)

$\Rightarrow \quad 21x - 19 = 5x + 6$ (combine like terms)

$\Rightarrow \quad 21x \mathbf{- 5x} - 19 = 5x \mathbf{- 5x} + 6$ (subtract $5x$ from each side)

$\Rightarrow \quad 16x - 19 = 6$ (simplify)

$$\Rightarrow \quad 16x - 19 + \mathbf{19} = 6 + \mathbf{19} \qquad \text{(add 19 to each side)}$$

$$\Rightarrow \quad 16x = 25 \qquad \text{(simplify)}$$

$$\Rightarrow \quad \frac{16x}{\mathbf{16}} = \frac{25}{\mathbf{16}} \qquad \text{(divide each side by 16)}$$

$$\Rightarrow \quad \boxed{x = \frac{25}{16}} \qquad \text{(simplify)}$$

Homework

23. Solve each equation:

a. $-4(a - 6) + (-5a - 3) = 6(2a + 1) - (5a + 4)$

b. $2(-8e - 6) - 8(-3e - 2) = 3(-8e - 7) - 4(-2e + 9)$

c. $5(-9r - 5) + 3(8r + 3) = -2(8r - 3) - 3(7r + 7)$

d. $9(-7j - 6) - 7(-5j + 3) = 6(8j + 1) + 5(5j - 8)$

e. $-6(-9d + 6) + 3(-3d - 9) = -8(-d - 9) - 8(-3d - 4)$

<u>Solve for x:</u> $\boldsymbol{\dfrac{nx - w}{y + z} = e - f}$

<u>Solution:</u> Notice the use of parentheses in the solution.

$$\frac{nx - w}{y + z} = e - f \qquad \text{(original formula)}$$

$$\Rightarrow \quad \frac{nx - w}{y + z}(y + z) = (e - f)(y + z) \qquad \text{(multiply each side by } y + z\text{)}$$

$$\Rightarrow \quad nx - w = (e - f)(y + z) \qquad \text{(simplify)}$$

$$\Rightarrow \quad nx = (e-f)(y+z)+w \qquad \text{(add } w \text{ to each side)}$$

$$\Rightarrow \quad \boxed{x = \frac{(e-f)(y+z)+w}{n}} \qquad \text{(divide each side by } n\text{)}$$

Homework

24. Solve each formula for x:

a. $x - c = d$ b. $2x + b = R$ c. $abx = c$

d. $\frac{x}{u} = N$ e. $x(y+z) = a$ f. $\frac{x}{n} = c - d$

g. $\frac{x}{a+b} = m - n$ h. $\frac{x}{c-Q} = c + Q$ i. $\frac{x}{R} = a - b + c$

j. $x(b_1 + b_2) = A$ k. $\frac{x}{a} - e = m$ l. $\frac{x+a}{b} = y$

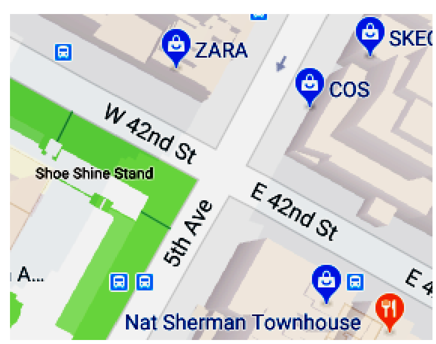
m. $\frac{ax - by}{c} = z$ n. $\frac{cx - a}{y+z} = h - g$ o. $\frac{ax+b}{c} - d = Q$

p. $\frac{9x + u - w}{Q + R} = m + n$ q. $9x - 7y + 13 = 0$

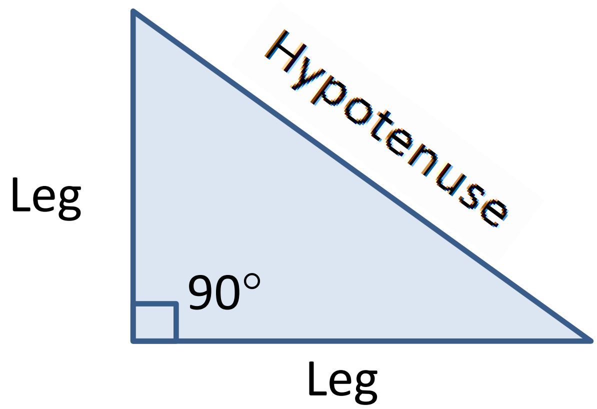
❑ THE PYTHAGOREAN THEOREM

The Right Triangle

An angle of 90° is called a ***right angle***, and when two things meet at a right angle, we say they are ***perpendicular***. For example, the angle between the floor and the wall is 90°, so the floor is perpendicular to the wall. And in Manhattan, 5th Avenue is perpendicular to 42nd Street.

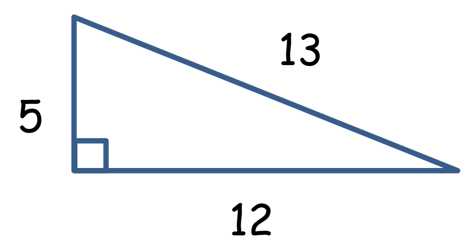


If a triangle has a 90° angle in it, we call it a ***right triangle***. The two sides that form the right angle (90°) are called the ***legs*** of the right triangle, and the side opposite the right angle is called the ***hypotenuse*** (accent on the 2nd syllable). It also turns out that the hypotenuse is always the longest side of a right triangle.



The Pythagorean Theorem

Ancient civilizations discovered that a triangle with sides 5, 12, and 13 would actually be a right triangle; that is, a triangle with a 90° angle in it.
[By the way, is it obvious that the hypotenuse must be the side of length 13?]



A Classic Right Triangle

But what if just the two legs are known? Is there a way to calculate the length of the hypotenuse? The answer is yes, and the formula dates back to 600 BC, the time of Pythagoras and his faithful followers.

To discover this formula, let's rewrite the three sides of the above triangle:

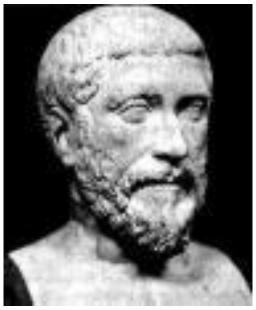
$$leg = 5 \qquad leg = 12 \qquad hypotenuse = 13$$

Here's the secret: Use the idea of <u>squaring</u>. If we square the 5, the 12, and the 13, we get 25, 144, and 169; that is,

$$5^2 = 25 \qquad 12^2 = 144 \qquad 13^2 = 169$$

and we notice that the <u>sum</u> of 25 and 144 is 169:

$$25 + 144 = 169$$



In other words, a triangle with sides 5, 12, and 13 forms a right triangle precisely because

$$5^2 + 12^2 = 13^2$$

Now let's try to express this relationship in words — it appears that

When you square the legs of a right triangle and add them together, you get the square of the hypotenuse.

As a formula, we can state it this way:

If a and b are the legs of a right triangle and c is the hypotenuse, then

$$a^2 + b^2 = c^2$$

We may have named the formula after Pythagoras, but the Babylonians were using the rule 1000 years before Pythagoras was born.

Solving Right Triangles

<u>EXAMPLE</u>: **The legs of a right triangle are 6 and 8. Find the hypotenuse.**

<u>Solution</u>: We begin by writing the Pythagorean Theorem. Then we plug in the known values, and finally determine the hypotenuse of the triangle.

$a^2 + b^2 = c^2$ (the Pythagorean Theorem)

$\mathbf{6}^2 + \mathbf{8}^2 = c^2$ (substitute the known values)

$36 + 64 = c^2$ (square each leg)

$100 = c^2$ (simplify)

What number, when squared, results in 100? A little experimentation yields the solution 10 (since $10^2 = 100$). Also be sure you can use your calculator to calculate $\sqrt{100}$. Our conclusion:

The hypotenuse is 10

Note: The equation $100 = c^2$ also has the solution $c = -10$ [since $(-10)^2 = 100$]. But a negative length makes no sense, so we stick with the positive solution, $c = 10$.

Homework

25. In each problem, the two legs of a right triangle are given. Find the **hypotenuse**.

a. 3, 4	b. 5, 12	c. 10, 24	d. 30, 16
e. 7, 24	f. 12, 16	g. 30, 40	h. 9, 40
i. 12, 35	j. 20, 21	k. 48, 55	l. 13, 84
m. 17, 144	n. 11, 60	o. 51, 140	p. 24, 143

❑ *INEQUALITIES*

You must score *between* 80% and 89% to get a B in your math class.

You must be *at least* 18 years of age to vote.

You can be *no taller* than 48 inches to play in the park.

These are all examples of quantities being greater than something or less than something. Since they are not equalities, they are called ***inequalities***.

We know that 5 is bigger than 3, which we can write as "5 > 3." The symbol ">" can also be read as "is larger than" or "is greater than."

But, of course, the fact that 5 is larger than 3 is the same as the fact that 3 is less than 5. This is written "3 < 5."

> means "is greater than"

< means "is less than"

≥ means "is greater than or equal to"

≤ means "is less than or equal to"

The symbol "≥" can be read "is greater than or equal to." For example, $9 \geq 7$ because 9 is indeed greater than or equal to 7. (Actually, it's greater than 7, but that doesn't change the fact that it's greater than or equal to 7.) And believe it or not, $12 \geq 12$ is a true statement — after all, since $12 = 12$, it's certainly the case that 12 is greater than or equal to 12.

The symbol "≤" is read "less than or equal to." A couple of examples are $6 \leq 10$ and $8 \leq 8$.

Homework

26. T/F:

a. $7 > 3$ b. $-2 < 1$

c. $13 \geq 13$ d. $-9 \leq -9$

e. $12 \geq 9$ f. $-18 \leq -20$

g. $\pi > 0$ h. $-\sqrt{2} \leq 0$

27. Express each statement as an inequality:

a. Your age, a, must be at least 18 years.

b. Your height, h, can be no taller than 48 inches.

c. Your years of experience, y, must exceed 10 years.

d. The number of driving tickets, t, must be fewer than 5.

e. The mean, μ (Greek letter mu), must be at least 75.

f. The standard deviation, σ (Greek letter sigma), must be no more than 10.

g. The energy, E, must be greater than 100.

h. The mass, m, must be less than 3.7.

Solutions

1. a. It's a decimal.

b. It can be written –99.0, a decimal.

c. It's approximately 1.047198, a decimal.

d. It's approximately 6.6332, a decimal.

e. It's 5, which is the decimal 5.0.

f. It equals 0.8, a decimal.

g. It's a (repeating) decimal, 0.232323 . . .

h. It's a (non-repeating) decimal.

i. It's a (terminating) decimal (or it's repeating with zeros).

j. It's a (non-terminating) decimal.

k. 0 = 0.0, a decimal.

l. It equals –4, which equals –4.0.

2. a. No real number squared is going to be negative.
b. No real number squared is going to be negative.
c. Any real number to the 4th power is going to be 0 or higher, never negative.

3. a. -17 b. 0 c. 3.5 d. -8π e. $\sqrt{2}$

4. a. True b. 0 c. positive d. negative e. 0

5. a. -9 b. 3 c. 0 d. $-\pi$ e. $\sqrt{2}$

6. a. $\frac{1}{5}$ b. $\frac{9}{2}$ c. $-\frac{3}{7}$ d. 1 e. Undefined f. π g. $\sqrt{3}$

7. a. False; 0 has no reciprocal. b. Undefined c. negative
d. positive e. 1

8. a. $\frac{1}{14}$ b. $\frac{3}{2}$ c. $-\frac{1}{99}$ d. $-\frac{4}{5}$ e. Undefined

9. a. x^{11} b. y c. r^{20} d. $a^{12}b^{12}$
e. $\frac{p^4}{q^4}$ f. a^8 g. $a^3b^3w^3$ h. 3

10. a. $-\frac{13}{10}$ b. 0 c. $\frac{3}{2}$ d. $-\frac{2}{15}$ e. $\frac{41}{5}$
f. $-\frac{5}{3}$ g. $-\frac{7}{3}$ h. $\frac{1}{3}$ i. $-\frac{15}{28}$

11. a. $\frac{5}{12}$ b. -1 c. 1 d. $\frac{4}{9}$ e. $-\frac{1}{18}$
f. $-\frac{28}{3}$ g. 6 h. $-\frac{1}{10}$ i. $-\frac{32}{25}$

12. True

13. a. $\frac{4}{9}$ b. $-\frac{1}{8}$ c. $\frac{1}{81}$ d. $-\frac{14}{19}$
e. $-\frac{1}{32}$ f. $\frac{64}{729}$

14. a. $\frac{9}{10}$ b. $\frac{3}{4}$ c. $\frac{1}{2}$ d. $\frac{1}{3}$ e. $\frac{11}{12}$
f. Not a real number g. $\frac{16}{17}$ h. 0

15. a. 230 b. 31 c. 42 d. 46 e. 1 f. 0
g. 5 h. 47 i. 396 j. 2916 k. 97 l. 4096

16. a. $(x+y)^2 = (2+1)^2 = 3^2 = 9$
b. $x^2 + y^2 = 10^2 + 5^2 = 100 + 25 = 125$
c. 63 d. 96 e. 44

17. a. $\frac{0}{15} = 0$ since $0 \times 15 = 0$, and 0 is the only number that accomplishes this.

b. $\frac{32}{0}$ is undefined because any number times 0 is 0, never 32; thus NO number works.

c. $\frac{0}{0}$ is undefined because any number times 0 is 0; thus EVERY number works.

18. a. 0 b. 0 c. 0 d. 0 e. Undefined f. Undefined
g. Undefined h. Undefined i. Undefined j. Undefined
k. Undefined l. 0

19. a. **20**. b. **21**. b.

22. a. You can't conclude anything — it depends on what's on the bottom. If the bottom is a non-zero number (like 7), then $\frac{0}{7} = 0$. If the bottom is zero, then $\frac{0}{0}$ is undefined.

b. This time we can conclude that the fraction is undefined, since division by 0 is undefined, no matter what's on the top of the fraction.

23. a. $a = \frac{19}{16}$ b. $e = -\frac{61}{24}$ c. $r = \frac{1}{16}$
d. $j = -\frac{41}{101}$ e. $d = \frac{167}{13}$

24. a. $x = d + c$ b. $x = \frac{R-b}{2}$ c. $x = \frac{c}{ab}$

d. $x = Nu$ e. $x = \frac{a}{y+z}$ f. $x = n(c-d)$

g. $x = (m-n)(a+b)$ h. $x = (c+Q)(c-Q)$ i. $x = R(a-b+c)$

j. $x = \frac{A}{b_1+b_2}$ k. $x = a(m+e)$ l. $x = by - a$

m. $x = \frac{cz+by}{a}$ n. $x = \frac{(h-g)(y+z)+a}{c}$

o. $x = \frac{c(Q+d)-b}{a}$ p. $x = \frac{(m+n)(Q+R)+w-u}{9}$

q. $x = \frac{7y-13}{9}$

25. a. 5 b. 13 c. 26 d. 34 e. 25 f. 20
g. 50 h. 41 i. 37 j. 29 k. 73 l. 85
m. 145 n. 61 o. 149 p. 145

26. a. T b. T c. T d. T
e. T f. F g. T h. T

27. a. $a \geq 18$ b. $h \leq 48$ c. $y > 10$ d. $t < 5$
e. $\mu \geq 75$ f. $\sigma \leq 10$ g. $E > 100$ h. $m < 3.7$

“The greatest mistake
you can make in life
is to be continually
fearing you will
make one.”

– **Elbert Hubbard (1856–1915)**